Analytical Model for Punching Load Prediction

Dr sc. tech. Ph. Menétrey
menetrey@ebbe.emchberger.ch
Emch+Berger AG Bern
Gartenstrasse 1
3001 Bern, Switzerland

ABSTRACT

The proposed model is based on the fact that the punching strength is controlled by the tensile stress in concrete along the inclined punching crack (as demonstrated with numerical simulation). Consequently, the punching strength can be obtained by integrating the vertical tensile stresses around the punching crack. The contribution of the reinforcements is taken into account similarly, by adding the vertical tensile forces of each reinforcement crossing the punching crack (flexural, shear reinforcement and prestress tendons). The transition between punching and flexural failure is expressed in terms of the inclination of the punching crack. The prediction of the punching load is successfully compared to the experimental results included in a databank.

Key words: punching failure, analytical model, shear reinforcement, databank

1 INTRODUCTION

The punching failure of reinforced concrete slabs supported on columns occurs when a conical plug of concrete suddenly perforates the slab above the column. As this failure mechanism is brittle-occurring without any warning sign and with a high reduction of the load carrying capacity, various constructive methods were developed to avoid it. Different geometry were considered with thicker slab or column enlargements and different types of reinforcements were set such as flexural reinforcements, bent-bars, studs, stirrups, shear-heads and prestressed tendons. Consequently, in order to predict the punching strength of all these different slabs, the developed analytical model should be based on simple and realistic concepts.

The general model developed by Menétrey [2] is based on simple and realistic concepts, so that the punching load prediction of various slabs is possible. The analytical expressions are based on results obtained using the numerical simulation of the failure mechanism with the computational tool developed by Menétrey et al.[6]. The influence of the flexural strength is linked to the punching strength with the punching crack inclination as proposed by Menétrey [3]. Finally, the analytical expression is enhanced with the treatment of the contribution of the stirrups and shear reinforcements as developed by Menétrey [4]. The resulting model is a unified and general model for punching load prediction. The prediction capabilities are discussed based on both databases developed by the *fib* task-group 4.3.

2 GENERAL ANALYTICAL MODEL

2.1 Integration of the tensile force

The punching strength is computed by integrating the vertical components of the concrete tensile stresses along the punching crack. A generalization of this model is obtained by adding the vertical projection of the forces in all the reinforcement, which are crossing the punching crack. The punching strength of a general slab is finally expressed as:

$$V_{pun} = V_c + V_{dow} + V_{sw} + V_{prt} \qquad (1)$$

where V_c is the concrete strength, V_{dow} is the resistant dowel-contribution due to the flexural reinforcement, V_{sw} is the vertical force in the studs, stirrups or bent-up bars which are well-anchored and Vprt is the vertical projection of the force in the tendons. The strength of a slab is consequently computed in a unified manner by summation of all the vertical tensile forces crossing the punching crack as illustrated in fig. 1.

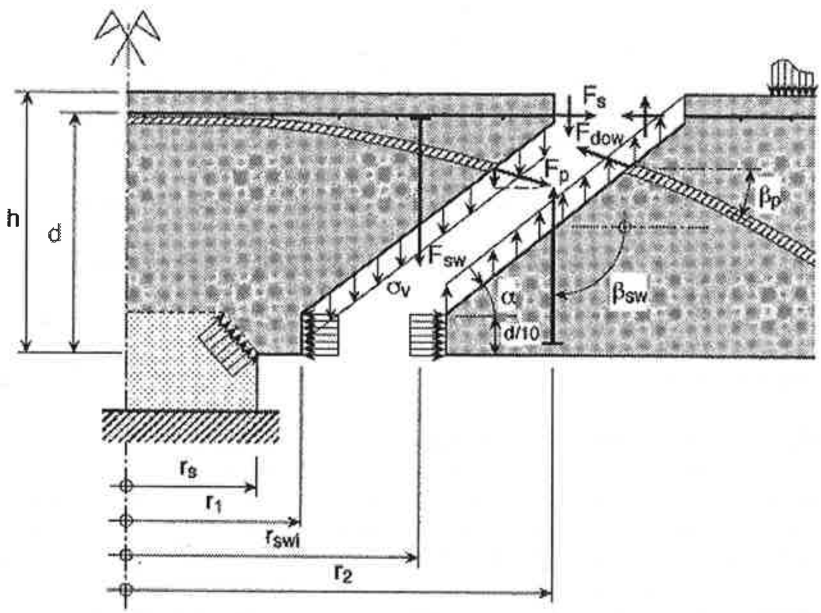

Figure1: Representation of the punching strength of a general reinforced slab

The punching failure is a sudden failure characterized by a rapid decrease of the load carrying capacity. However, the punching crack causing the punching failure is not formed suddenly, but is preceded by the formation of internal micro-cracks. This micro-crack formation is reported experimentally by Moe [7] and Regan [8]. Along the same line, Menétrey et al. [6] showed that punching failure results from a coalescence of micro-cracks (progressive phase) followed by a crack propagation (sudden phase). Because the micro-crack formation is progressive, steel resistance is gradually activated and the force in the reinforcement can be added to the concrete strength.

2.2 Concrete strength

The punching strength of concrete is determined by integrating the vertical tensile stresses σ_v around the punching crack (truncated cone in shape) comprised between two r_1 and r_2 as shown in fig. 1 which are:

$$r_1 = r_s + \frac{1}{10}\frac{d}{tan\alpha}; \quad r_2 = r_s + \frac{d}{tan\alpha} \tag{2}$$

where r_s is the radius of the column and α the punching crack inclination. Experimentally, the punching crack inclination varies from 25° to 35° and the value α=30°allows the best prediction. The surface between r_1 and r_2 is: $\pi(r_1 + r_2)$ s$ where s is the inclined length expressed as:

$$s = \sqrt{(r_2 - r_1)^2 + (0.9\,d)^2} \tag{3}$$

Around the punching crack, a constant tensile stress distribution is assumed for simplicity. The analytical expression to compute the punching strength of concrete is consequently expressed as follows:

$$V_c = \pi(r_1 + r_2)\,s\,\sigma_v = \pi(r_1 + r_2)\,s\,f_t^{2/3}\,\xi\eta\mu \tag{4}$$

in terms of the concrete tensile strength f_t. The influence of the percentage of reinforcement ρ on the tensile stresses was determined with numerical simulations and is given by:

$$\xi = \begin{cases} -0.1\rho^2 + 0.46\rho + 0.35 & 0 < \rho < 2\% \\ 0.8 & \rho \geq 2\% \end{cases} \tag{5}$$

The size-effect laws obtained numerically are combined and included in the developed analytical model with the parameter:

$$\mu = 1.6\,(1 + d/d_a)^{-1/2} \text{ with } \mu < 1.2 \tag{6}$$

where d_a is the maximum aggregate size.

This size-effect law was derived for a constant ratio h/r_s=2 where h is the slab thickness. However, as reported experimentally, the radius of the column or similarly the radius where the punching crack is initiated influences the size-effect law. This phenomenon was reproduced numerically and an analytical relation is proposed:

$$\eta = \begin{cases} 0.1(r_s/h)^2 - 0.5(r_s/h) + 1.25 & 0 < r_s/h < 2.5 \\ 0.625 & r_s/h \geq 2.5 \end{cases} \tag{7}$$

This parameter is determinant to predict the punching strength of slabs with shearhead or shear reinforcement for which the punching crack is located outside the reinforcement (as illustrated in fig. 2b for slabs with studs for which r_{sc} should replace r_s in eq. 7) as the radius of the punching crack initiation is large.

2.3 Dowel-effect contribution

The dowel-effect is a shear transferred by reinforcing bars crossing the punching crack. This mechanism increases the punching load significantly for slabs with orthogonal reinforcement. Regan and Braestrup [10] reported that up to 34% of the total punching load of slabs with orthogonal reinforcement can be attributable to dowel-effect. Consequently, the prediction of the experimental punching load for slabs with orthogonal reinforcement should consider the dowel-effect.

The shear force which can be transferred by reinforcing bars crossing the punching crack is computed so that:

$$V_{dow} = \frac{1}{2}\sum^{bars} \Phi_s^{\,2} \sqrt{f_c\, f_s\,(1-\zeta^2)}\,\sin\alpha \qquad (8)$$

where the summation is performed for all bars crossing the punching crack and ϕ_s is the diameter of the corresponding bars. A parabolic interaction is assumed between the axial force and the dowel force in the reinforcing bar which is expressed with the term: $(1-\zeta^2)$ where $z = \sigma_s/f_s$ and σ_s is the axial tensile stress in the reinforcing bar (see eq. 1) characterized by a yield strength f_s. The tensile stress σ_s is obtained by projection of the force in the compressive strut (illustrated in fig. 1): $V_{pun}/\sin\alpha$ in the horizontal reinforcement which gives $V_{pun}/\tan\alpha$ and by dividing by the sum of the area of the reinforcing bars crossing the punching crack so that:

$$\sigma_s = \frac{V_{pun}/\tan\alpha}{\sum^{bars} A_s} \qquad (9)$$

The dowel contribution in eq. 8 is reduced with the term $\sin\alpha$, to take into account the angle between the flexural reinforcing bar and the punching crack (in a vertical plane). In eq. 8, the factor 1/2 gives the best approximation, because the reinforcing bars are not crossing the punching crack at a right angle (in a horizontal plane).

2.4 Shear reinforcement contribution

Shear reinforcements such as studs, stirrups, bent-bars or bolts are used in order to increase the failure load and to reduce the sudden decrease of the load carrying capacity. The computation of the failure load in such slabs should differentiate the following failure mechanisms (illustrated in fig. 2 for a slab with studs).

1. A failure mechanism for which the punching crack is located in between the column face and the first row of shear reinforcement. The computation of the corresponding failure load must consider the interaction between the punching V_{pun} and the flexural strength V_{flex} in terms of the punching crack inclination $\alpha_a = \arctan((r_{swi}-\rho s)/d)$ as given in eq. 13.

2. A failure mechanism for which the punching crack is initiated outside the last row of shear reinforcement. The punching strength is computed similarly to the one of a normal reinforced concrete slab except that instead of the radius of the column r_s, the radius of punching crack initiation r_{sc} should be considered. The influence of this radius on the size-effect is treated with the parameter η given in eq. 7.

3. A failure mechanism for which the punching crack crosses the shear reinforcement.

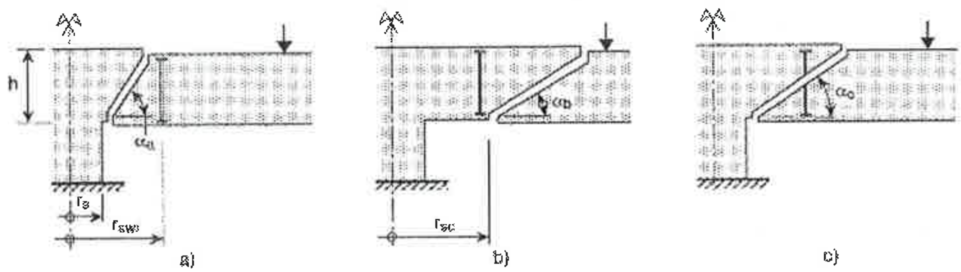

Figure 2: Three punching failure mechanisms: (a) punching crack located in-between the column face and the first row of studs, (b) punching crack outside the last row of studs, and (c) punching crack crossing the studs

The contribution to the punching strength of shear reinforcements (studs, stirrups, bent-bars or strengthening bolts) for mechanism c) are differentiated according to their bond properties: (1) shear reinforcements made with plain bars and anchorage denoted as ***studs***, (2) shear reinforcements made with high-bond bars denoted as ***stirrups***. The contribution of injected strengthening bolts (set after perforating the slab) is determined similarly to the contribution of studs or stirrups according to their bond properties. As a special case, for non-injected strengthening bolts, the concrete and the shear reinforcements contributions cannot be added because they do not interact as established by Menétrey and Brühwiler [5]. The contribution of the shear reinforcements to the punching strength is computed by summing each reinforcement contribution (in different rows, made with different types of reinforcement) crossing the punching crack. The shear reinforcement are characterized by a cross-section A_{sw} and a Young's modulus E_{sw}.

The failure mechanism for which the punching crack crosses the studs is initiated by micro-cracks. Due to micro-cracking, the slab thickness increases, resulting in the loading of the studs. Consequently, the studs sustain a so-called displacement control loading for which the displacement corresponds to the summation of the micro-cracks opening between both studs extremities. The maximum force in the stud is expressed as:

$$V_{sw} = \sum A_{sw} E_{sw} \frac{5 G_f}{f_t l \cdot \cos\alpha} \sin(\beta_{sw}) < \sum A_{sw} f_{sw} \sin(\beta_{sw}) \qquad \text{for studs} \qquad (10)$$

where G_f is the concrete fracture energy.

In the opposite, the tensile force in the stirrups is transmitted by bond stresses to concrete over the transmission length (defined as the length over which slip between steel and concrete occurs). If this length is available, the yield strength of the stirrups is reached so that:

$$V_{sw} = \sum A_{sw} f_{sw} \sin(\beta_{sw}) \qquad \text{for stirrups} \qquad (11)$$

If this length is not available, the carrying force in the stirrup is function of the anchorage of the stirrup's extremity. Consequently, the anchorage of the stirrups has to be carefully set, specially in the top face of slabs (positive bending) and for short stirrups.

2.5 Prestressing tendons contribution

The punching strength of a slab reinforced with inclined prestressing tendons is enhanced by adding the vertical projection of the tendon forces. This vertical projection of the tendons force V_{prt} is

$$V_{prt} = \sum^{tendons} A_p \sigma_p \sin(\beta_p) \tag{12}$$

where the summation is performed for the prestressing tendons crossing the punching crack, A_p is the area of the prestressing tendons, σ_p is the tensile stress of the prestressing steel, and β_p is the inclination of the tendons at the intersection of the punching crack with the plane of the slab as shown in fig. 1.

3 PUNCHING AND FLEXURAL STRENGTH

The geometry of the failure surface plays an important role, especially when the failure surface is forced into a shape different from that giving the normal punching resistance as suggested by Regan [9]. The influence of the punching crack inclination was presented by Menétrey [3] based on experimental results. It is shown that the punching crack inclination α allows to link both flexural and punching failure loads. The proposed analytical expression is considered here:

$$V_{fail} = V_{pun} + (V_{flex} - V_{pun}) \left\{ \sin(\frac{3}{2}\alpha - 45°) \right\}^{1/2} \quad \text{with } 30° \le \alpha \le 90° \tag{13}$$

It can be noted that for a punching crack inclination $\alpha = 30°$ then $V_{fail} = V_{pun}$ and for $\alpha = 90°$ then $V_{fail} = V_{flex}$. For design application, the flexural failure load V_{flex} can be determined with the yield-line theory as presented by Gesund and Kaushik [1].

4 COMPARISON WITH RESULTS IN A DATABANK

The predictions of the model are compared with the experimental data included in the database developed by the *fib* task group 4-3. For the prediction of the punching load of slabs without shear reinforcement only the terms V_c and V_{dow} are activate according to eq. 1 Due to the amount of tests and required data, some simplifications were made: (1) The punching crack inclination was assumed to be always 30°, (2) The data were taken as they were given without any special check. (3) Some data like the aggregate size and the diameter of the flexural reinforcement were missing for various tests. In order to pursue with a database with many tests, the data were completed and the aggregate size was assumed to be $d_a = 20$ mm and the diameter of the flexural reinforcement was assumed to be $\phi_s = 10$ mm. (4) No distinctions were made between circular and square slabs or columns (treated as circular ones). The comparison of the predicted and experimental results are shown in fig. 3. The thick line shown the polynomial regression, which is very close to the ideal prediction line (dotted line).

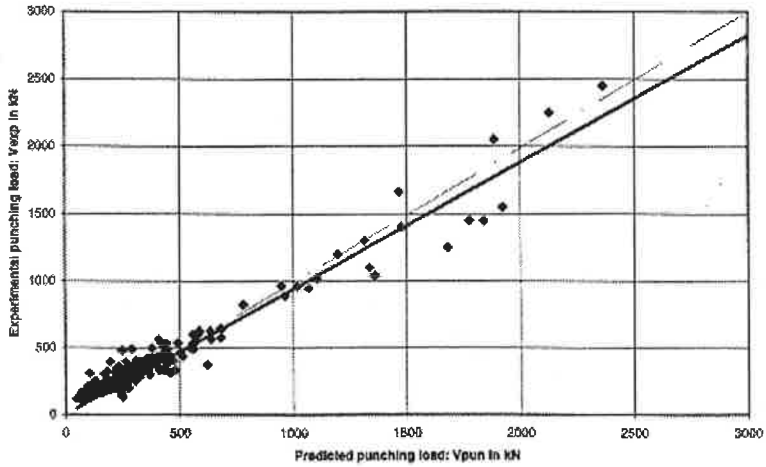

Figure 3: Punching load prediction in slab without shear reinforcement

The prediction of the punching load for slabs with shear reinforcements was performed by adding the contribution V_{sw}. The differences between stirrups and studs as presented in sec. 2.4 is not considered, because the concrete fracture energy necessary to express this difference was almost never reported experimentally. Therefore, eq. 11 was considered, were the full strength of the shear reinforcement is activated. The prediction of the punching load in three sections was performed as illustrated in fig. 2. The predicted failure load and the experimental one are shown in fig. 4 for the three mechanisms. It can be seen that the dispersion of the prediction is large.

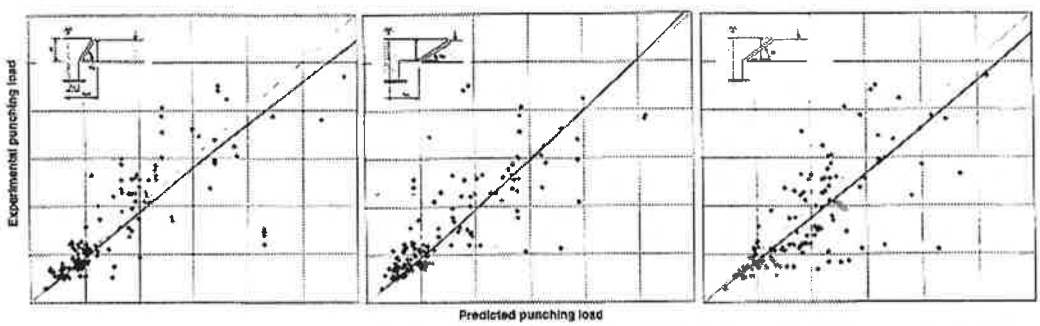

Figure 4: Failure load in slab with shear reinforcement for the three mechanisms

However, the punching load prediction should consider the minimum value of the ones obtained with the three mechanisms and this value is plotted in fig. 5. It can be seen that the dispersion is mainly reduced. Furthermore, the prediction is not diffuse on the overall area, but is restricted to the area on the safety side. These justify that the failure load should be computed for the three mechanisms, and the minimum value should be considered.

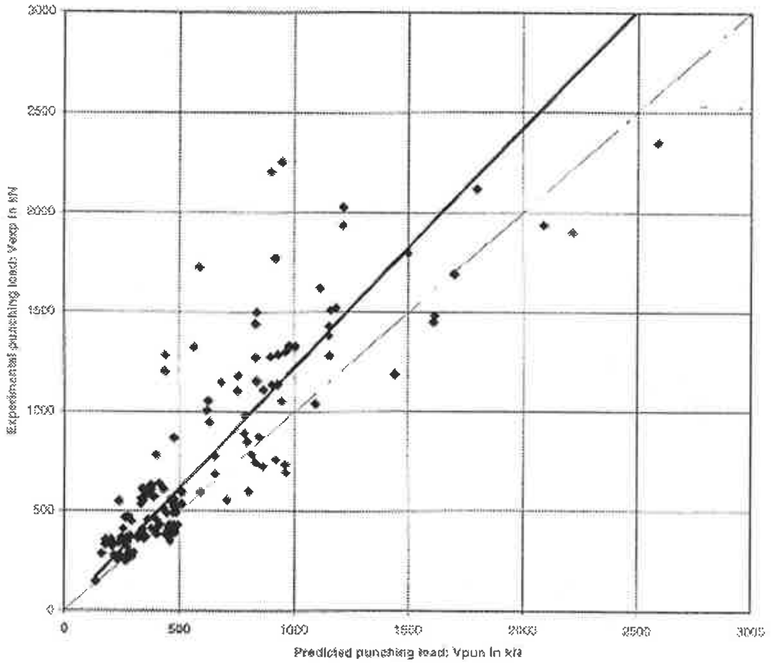

Figure 5: Punching load prediction in slab with shear reinforcement (minimum value)

5 CONCLUSION

An analytical model to predict the punching failure load which is based on the integration of the vertical tensile stresses around the punching crack has been described. The strength of the reinforcement is treated in a unified approach by adding the vertical tensile forces in each type of reinforcement, so that the dowel force, the force in stirrups, and the prestressing tendons contributions are taken into account. The influence of the punching crack inclination is considered by linking the flexural failure load. The predicted failure load is sucessfully compared with the experimental results available in the database developed by the *fib* task-group 4.3.

This analytical method can be easily adapted for design purposes if the following are satisfied: (1) safety factors should be included, (2) the dowel-effect should be neglected because large deflections are necessary, and (3) the analytical expression should be extended for real slabs by taking into account holes or stiffening.

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